Coriolis Effects on Görtler Vortices in the Boundary-Layer Flow on Concave Wall

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I. Introduction

FOR many years, hydrodynamic instabilities have been the subject of intense research activity, in which scientists from different areas (applied mathematics, mechanics, physics) have been involved. In fact, the wide variety of problems arising from the modeling of hydrodynamic instabilities in experiment and in computers are exciting for fundamental research and promising for technological applications.

One of those problems is the Taylor-Görtler centrifugal instability that occurs in the boundary-layer flow over a concave wall. This problem has been investigated since Görtler first studied it in 1940[1] and remains challenging because of the complexity of the base flow.[2] One of the many difficulties occurring in the theoretical study of the Taylor-Görtler vortices is that the separation of the spatial variables is only valid for very large wave numbers[3] that are never reached in experimental or practical situations. In fact, the problem is a Cauchy problem in streamwise coordinate. Aihara[4] has developed an integral method that can be used in order to obtain qualitatively general results and has reproduced the known results obtained with numerical resolution.[5]

The Taylor-Görtler vortices, with their three-dimensional nature, change flow properties (wall friction, separation characteristics), and so it is important when engineers design a practical curved plate in a given flow to ensure the laminar flow control (LFC). They also strongly increase the heat transfer in the boundary layer on cooled walls as in turbine blades. Moreover, the boundary layers developed on curved blades in turbomachineries are subjected to rotation; therefore, it is important to determine how the stability of the boundary layer is modified.

In this Note, we study the stability of the boundary layer on a concave wall subject to uniform rotation around the axis of a curved plate. The rotating curved plates are encountered in practical problems such as the turbine blades of turbomachineries, pulp flow in paper refineries, flow in centrifugal pumps, and centrifugal chemical reactors. The problem of stability of a rotating boundary layer over a flat plate has been investigated numerically by Potter and Chawla.[6] They have shown that rotation induces perturbations with spanwise dependence in the flow. We show that, when the rotating boundary layer on the concave wall with rotation axis parallel to the shear vorticity becomes unstable to longitudinal vortices, the rotation in opposite direction of the shear vorticity stabilizes the flow and delays the appearance of Taylor-Görtler vortices. We apply the integral method, used already by Aihara for the Taylor-Görtler instability,[4] and we assume the constant boundary-layer thickness, i.e., it does not grow in the longitudinal direction. Hence, the perturbative flowfield will be assumed for simplicity to be independent of the longitudinal coordinate. According to Hall,[3] the assumption of constant boundary-layer thickness leads to results that are valid only for large wave numbers (small wavelengths compared to the boundary-layer thickness). However, once we are interested in the effects of the rotation on Taylor-Görtler instability, we adopt such an assumption to simplify the analysis. In order to justify this approximation, the last part of the present work will be concerned with the asymptotic behavior of the marginal stability curve for large wave numbers.

In the next section, we derive the governing equations of the perturbed flow over a concave plate in rotating frame of reference. Section III contains results and their analysis. The last section addresses concluding remarks.

II. Problem Formulation and Basic Equations

Let us consider a flow over a slightly concave wall (i.e., its radius of curvature R is very large compared to the momentum thickness θ). We shall adopt Cartesian coordinates (x^*, y^*, z^*), with x^* in the mainstream direction, y^* normal to the surface, and z^* transverse to the (x^*, y^*) plane. For our calculations, we neglect the streamwise variation of the field and we suppose that the wall is rotating around the z^* axis with a constant angular velocity $\Omega = (0, 0, \Omega)$, as it can be seen in Fig. 1. In a relative frame fixed on the surface, the conservation equations of mass and momentum for an incompressible fluid can be written as follows:

$$\operatorname{div} \boldsymbol{V}^* = \boldsymbol{0} \tag{1a}$$

$$\rho\left(\frac{\partial \boldsymbol{V}^*}{\partial t^*} + (\boldsymbol{V}^* \cdot \nabla)\boldsymbol{V}^*\right) = -\nabla P^* + \mu\Delta \boldsymbol{V}^* - 2\rho\Omega \boldsymbol{V}^* \tag{1b}$$

with the boundary conditions

$$\boldsymbol{V}(z^* = 0) = \boldsymbol{0} \tag{2a}$$

$$\boldsymbol{V}^* \to \boldsymbol{U}_0^* = (U_0, 0, 0) \quad \text{when} \quad z^* \to +\infty \tag{2b}$$

where starred quantities are dimensional and the pressure gradient includes centrifugal terms. Time, spatial coordinates, velocity, and pressure are scaled respectively by θ/U_0, θ, U_0, ρU_0^2, and then Eqs. (1) and (2) become

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3a}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} - \frac{\theta}{R}uv + w\frac{\partial u}{\partial z} = \frac{1}{Re_\theta}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + Ro\frac{\theta}{R}v \tag{3b}$$

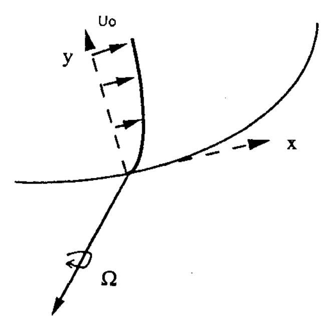

Fig. 1 Schematic representation of the concave wall and the rotation vector Ω.

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$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{\theta}{R}u^2$$

$$=-\frac{\partial p}{\partial y}+\frac{1}{Re_\theta}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-Ro\frac{\theta}{R}u \tag{3c}$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{Re_\theta}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{3d}$$

We have introduced the Reynolds number $Re_\theta = \rho U_0\theta/\mu$ based on the freestream velocity U_0 with μ as the kinematic viscosity and $Ro = 2\Omega R/U_0$ the rotation number, which is the ratio of inertial to Coriolis forces . This definition is adopted as in Ref. 7 because it is a global quantity independent of local streamwise coordinates, and it has the advantage of eliminating few inconsistencies in dimensionless equations. The rotation does not affect the base flow velocity but it modifies the normal to wall pressure gradient. We go further, following classical stability methods, by putting the velocity and the pressure as a sum of the base and disturbance flows as

$$u(y,z,t)=U(y)+u'(y,z,t)=U(y)+\tilde{u}(t)\hat{u}(y,z)$$

$$(v,w)=\{v'(y,z,t),w'(y,z,t)\}=\{\tilde{v}(t)\hat{v}(y,z),\tilde{w}(t)\hat{w}(y,z)\}$$

$$p=P(y)+p'(y,z,t)$$

Following Aihara,[4] we insert these expressions into Eqs. (3) and deduce the energy and vorticity equations of the perturbed flow, which lead to two simultaneous nonlinear differential equations for amplitudes $\tilde{u}(t)$ and $\tilde{v}(t)$:

$$\tilde{u}\frac{d\tilde{u}}{dt}+A_1\tilde{v}\frac{d\tilde{v}}{dt}+A_2\tilde{u}\tilde{v}+A_3\tilde{u}^2+A_4\tilde{v}^2+A_5\tilde{u}^2\tilde{v}=0 \tag{4a}$$

$$\frac{d\tilde{v}}{dt}+B_1\tilde{u}+B_2\tilde{v}+B_3\tilde{u}^2=0 \tag{4b}$$

where the coefficients are given by

$$A_1=\frac{1}{\langle\hat{u}^2\rangle}\left[\langle\hat{v}^2\rangle+\langle\hat{w}^2\rangle\left(\frac{\partial\hat{v}}{\partial y}\right)^2\left(\frac{\partial\hat{w}}{\partial z}\right)^{-2}\right]$$

$$B_1=\frac{\theta}{R}\frac{1}{\langle\xi^2\rangle}\left[2\left\langle\xi U\frac{\partial\hat{u}}{\partial z}\right\rangle+Ro\left\langle\xi\frac{\partial\hat{u}}{\partial z}\right\rangle\right]$$

$$A_2=\frac{1}{\langle\hat{u}^2\rangle}\left[\left\langle\left(\frac{dU}{dy}+Ro+\frac{2\theta}{R}U\right)\hat{u}\hat{v}\right\rangle\right]$$

$$B_2=-\frac{1}{Re_\theta\langle\xi^2\rangle}\left[\left\langle\xi\left(\frac{\partial^2\xi}{\partial y^2}+\frac{\partial^2\xi}{\partial z^2}\right)\right\rangle\right]$$

$$A_3=\frac{1}{Re_\theta\langle\hat{u}^2\rangle}\left[\left\langle\left(\frac{\partial\hat{u}}{\partial y}\right)^2\right\rangle+\left\langle\left(\frac{\partial\hat{u}}{\partial z}\right)^2\right\rangle\right]$$

$$B_3=2\frac{\theta}{R}\frac{1}{\langle\xi^2\rangle}\left[\left\langle\xi\hat{u}\frac{\partial\hat{u}}{\partial z}\right\rangle\right]$$

$$A_4=\frac{1}{Re_\theta\langle\hat{u}^2\rangle}\left\{\left\langle\left(\frac{\partial\hat{v}}{\partial y}\right)^2\right\rangle+\left\langle\left(\frac{\partial\hat{v}}{\partial z}\right)^2\right\rangle\right.$$
$$\left.+\left[\left\langle\left(\frac{\partial\hat{w}}{\partial y}\right)^2\right\rangle+\left\langle\left(\frac{\partial\hat{w}}{\partial z}\right)^2\right\rangle\right]\left(\frac{\partial\hat{v}}{\partial y}\right)^2\left(\frac{\partial\hat{w}}{\partial z}\right)^{-2}\right\}$$

$$A_5=\frac{\theta}{R}\frac{1}{\langle\hat{u}^2\rangle}\langle\hat{u}^2\hat{v}\rangle$$

The mean value is defined as

$$\langle\phi\rangle=\int_0^\lambda dz\int_0^{+\infty}\phi\,dy$$

where λ is the wavelength of the periodicity in the z direction ($\lambda = 2\pi/\alpha$). The quantity $\xi = (\partial\hat{w}/\partial y) - (\partial\hat{v}/\partial z)$ is the vorticity component of the disturbance in the (y, z) plane. The rotation effects are encountered normal to the coefficients B_1 and A_2. The time dependence of the perturbative velocity field can be sought in the form exp (σt), where σ represents the temporal growth rate of perturbations. In accordance with the experimental results on rotating curved channel flow for small Ro,[8] we assume the principle of exchange of stability to be valid and set $\sigma = 0$ for the marginal stability. We therefore obtain from Eqs. (4) the following relation for the marginal stability curve:

$$A_3B_2^2-A_2B_1B_2+A_4B_1^2=0 \tag{5}$$

This equation gives the following expression for the control parameter, i.e., the Görtler number $Go_\theta^2 = Re_\theta^2(\theta/R)$:

$$Go_\theta^2=-\frac{\left[\left\langle\left(\frac{\partial\hat{u}}{\partial y}\right)^2\right\rangle+\left\langle\left(\frac{\partial\hat{u}}{\partial z}\right)^2\right\rangle\right]\left[\left\langle\xi\left(\frac{\partial^2\xi}{\partial y^2}+\frac{\partial^2\xi}{\partial z^2}\right)\right\rangle\right]}{\left[\left\langle\hat{u}\hat{v}\left(\frac{dU}{dy}+Ro\right)\right\rangle\right]\left[2\left\langle\xi U\frac{\partial\hat{u}}{\partial z}\right\rangle+Ro\left\langle\xi\frac{\partial\hat{u}}{\partial z}\right\rangle\right]} \tag{6}$$

According to this relation, the rotation is involved in the transfer of energy from the mean shear and centrifugal motion to the perturbed flow, but it does not contribute to the dissipation processes of energy and vorticity. For the marginal stability curve, the coefficients of the nonlinear terms have been neglected because of small amplitude perturbations, i.e., $A_5 \approx B_3 \approx 0$.

III. Results and Analysis

In order to calculate the integral coefficients A_i and B_i, we expand the disturbance flow in Fourier series and retain only the fundamental modes with wave number α:

$$[\hat{u}(y,z),\hat{v}(y,z)]=[u_1(y),v_1(y)]\,df/dz$$

$$\hat{w}(y,z)=w_1(y)f(z),\qquad \text{with}\quad f(z)=\cos(\alpha z)$$

We choose, following Aihara, the structure perturbative functions in the form of piecewise functions given in Fig. 2 (this choice is justified for only for small $|Ro|$). In that case, calculations of the A_i and B_i coefficients can be achieved analytically, and we obtain the Görtler number expressed as a ratio of polynomials in α^2:

$$Go_\theta^2=\frac{a_3b_2^2}{b_1}\frac{1}{a_{20}b_2+(a_2b_2-a_4b_1)K}\frac{1}{1+CRo} \tag{7}$$

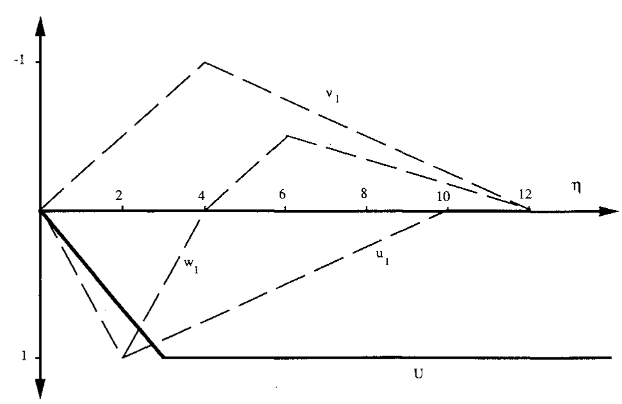

Fig. 2 Approximate velocity profile of the base flow and of the perturbation field in the boundary layer after Aihara, reasonable for $|Ro| \ll 1$.

where $K = 2\ (\theta/R)$ is the dimensionless curvature, and the other coefficients are given by

$$a_{20} = -\frac{11}{120\gamma} - \frac{159}{160}Ro, \qquad a_2 = -\frac{5149}{1920}, \qquad a_3 = \frac{3 + 16\gamma^2\alpha^2}{16\gamma^2}$$

$$a_4 = \frac{48\gamma^2\alpha^4 + 9\alpha^2 + 5}{40\gamma^2\alpha^2}, \qquad b_1 = -\frac{5149\gamma^2\alpha^4 + 594\gamma\alpha^2}{48(48\gamma^2\alpha^4 + 12\gamma\alpha^2 + 5)}$$

$$b_2 = \alpha^2, \qquad C = \frac{53/16\gamma\alpha^2 + 5/12}{5149/576\gamma\alpha^2 + 33/32}$$

with $\gamma = \delta_0/\theta = 1.51$, where δ_0 is the displacement boundary-layer thickness.

From Eq. (7), we found Aihara's result in the absence of rotation and in the limit of small curvature ($K = 0$), i.e, the critical state is given by $\alpha_c = 0$, $Go_c = 0.602$. However, this result is inconsistent as it gives the instability with infinite wavelength in the axial direction. After Hall,[3] this inconsistency is due to the separation of variables, which is only valid for large wave numbers ($\alpha \gg 1$), but it may be removed by including curvature factor K (Fig. 3a). For small curvature, the relation (7) shows that small positive Ro destabilizes the flow, whereas small negative Ro stabilizes the flow (Fig. 3b). For $Ro \approx -0.061$, the critical curve (Fig. 4) exhibits a singularity similar to that observed in curved rotating channel flow.[7,8] One should expect the existence of oscillatory modes in the vicinity of that point. The effect of the rotation on the stability of the flow is confirmed by many results about Dean's problem with rotation around a perpendicular axis to the mainstream flow.[7,8] In fact, applying the generalized Rayleigh stability criterion, one realizes that the potentially unstable region has larger extension for $Ro > 0$ then for $Ro < 0$. The curvature effect on Görtler instability has been investigated numerically by Herbert,[9] and our results are in the same direction.

In the limit of large wave numbers, our calculations give the following asymptotic behavior as already predicted by Hall[3]:

$$Go_\theta^2 \approx \alpha^4, \qquad \text{when} \quad \alpha \to +\infty$$

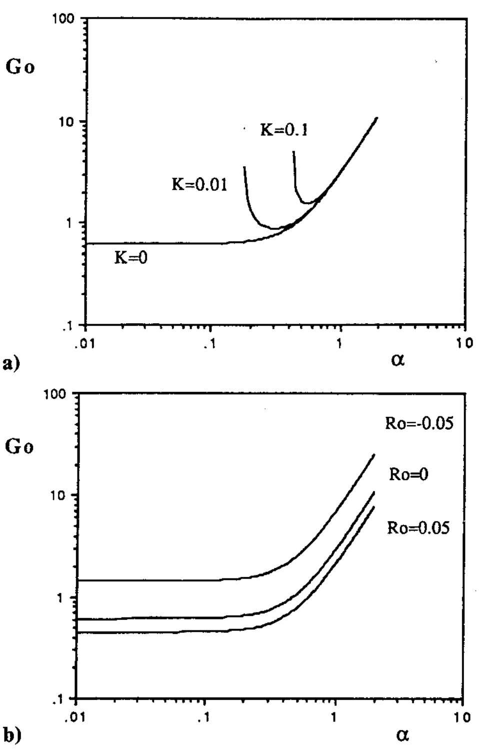

Fig. 3 Marginal stability curve for different values: a) curvature K ($Ro = 0$); b) rotation number $Ro(K = 0)$. The cases $Ro = 0$ and $K = 0$ correspond to the Aihara results.

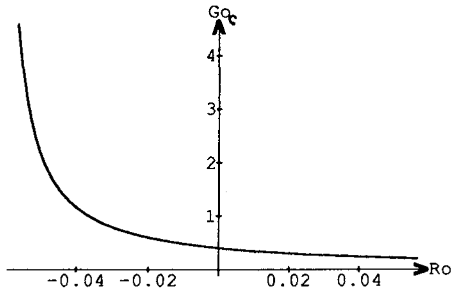

Fig. 4 Phase diagram (Ro, Go_c) for slowly rotating concave plates.

According to Hall,[3] for vortices having wavelength λ of the same order of magnitude as the momentum thickness θ, there is no justification for making the parallel flow approximation. For small vortices ($\alpha \gg 1$), the separation for the eigenfunctions can be made, and it is found that they obey the following system of equations:

$$(1 - \alpha^{-2}D^2)\, u_1 = -\alpha^{-2}\frac{dU}{dy}\, v_1 \tag{8a}$$

$$(1 - \alpha^{-2}D^2)^2 v_1 = -2\alpha^{-2}Go_\theta^2(U + Ro/2)u_1 \tag{8b}$$

where $D = d/dy$, with the boundaries conditions:

$$u_1 = v_1 = Dv_1 = 0, \qquad \text{when} \quad z = 0 \tag{9a}$$

$$u_1 = v_1 = Dv_1 = 0, \qquad \text{for} \quad z \to +\infty \tag{9b}$$

From Eq. (8a), we have $v_1/u_1 = O\,(\alpha^2)$, and by substitution in Eq. (8b), we find that the control parameter Go_θ^2 can be expressed at the first order, as follows:

$$Go_\theta^2 = \frac{\alpha^4}{2(U + \mathrm{R}o/2)} + \cdots$$

where it is assumed that $U = O(1)$. This short calculation tends to justify the asymptotic form of our neutral curves when $\alpha \gg 1$. At the same time, we show that for a slowly rotating concave plate, the base flow velocity profile can be replaced by the equivalent velocity profile $U + \mathrm{R}o/2$, as in Ref. 7.

IV. Conclusions

Using Aihara's method, we have qualitatively predicted the effect of a small amount of rotation of a concave surface on the instability of a flow past it. The rotation vector $\mathbf{\Omega}$ was supposed to have only one component Ω in the z direction (that means that the local vorticity has the same direction as the global one). It was found that, for $\Omega < 0$, the flow was stabilized by the rotation; at opposite for $\Omega > 0$, the rotation destabilizes the flow as it can be seen on the neutral curves. But this method is unable to give us any information about the flow behavior for nonsmall values of Ro; it cannot be used to calculate the oscillatory modes induced by the mutual effect of curvature and rotation obtained, for example, in rotating curved channel flow.[8] The components of rotation in the other directions may create mean flow equivalent to the general problem of crossflow instabilities, as in swept-wing flows.[10]

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Surface Temperature Effects on Boundary-Layer Transition

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Introduction

LINEAR stability theory provides guidance on the effects of surface temperature on boundary-layer stability. Cooling the surface stabilizes first-mode disturbances and destabilizes second-mode disturbances.[1,2] Therefore, for subsonic and low supersonic Mach numbers, where first-mode disturbances are the dominant disturbances, cooling the surface should increase the transition Reynolds number. For hypersonic Mach numbers, where second-mode disturbances are the major disturbances, cooling the surface should reduce the transition Reynolds number. Supersonic wind-tunnel transition results have generally been compatible with linear stability theory; however, the hypersonic transition data have produced conflicting results. Reducing the surface temperature in hypersonic transition experiments did not result in a consistent trend. Available data indicate trends of increasing, decreasing, or no change in transition Reynolds number with reductions in surface temperature. Figure 1 contains supersonic and hypersonic wind-tunnel data collected by Potter.[3] $(Re_{X_T})_{AD}$ is the transition Reynolds number obtained under adiabatic conditions, and M_e is the Mach number at the edge of the boundary layer. Surface cooling is seen to significantly increase the transition Reynolds number for the lower supersonic Mach numbers, with a smaller increase at hypersonic Mach numbers. The results of Sanator et al.[4] [not shown in Fig. 1 because the value of $(Re_{X_T})_{AD}$ was not known] at $M_e = 8.8$ indicated no significant change of transition Reynolds number on a sharp cone with changes of wall-to-stagnation temperature ratio from 0.08 to 0.37. Some additional data (not shown in Fig. 1) of Stetson and Rushton[5] at $M_\infty = 5.5$ and Mateer[6] at $M_\infty = 7.4$ indicate a reduction in the transition Reynolds number on cone models with a reduction in the temperature ratio. Thus, the experiments that showed the expected reduction in hypersonic transition Reynolds number with a reduction in surface temperature seemed to be more of an exception rather than the general case.

Contents

Recent boundary-layer stability experiments[7] at a freestream Mach number of 8, along with prior stability experiments of Kendall[8] and Demetriades,[9] have provided a likely explanation for some of the confusion regarding surface temperature effects on hypersonic transition data. Hot-wire anemometry experiments have provided many details of the major disturbances found in laminar boundary layers within a wind-tunnel environment. The major disturbances in the laminar boundary layer of a sharp cone at zero angle of attack at Mach 8 were second-mode disturbances (the high-frequency, acoustical-type disturbances identified by Mack's linear stability analyses[1,2]). Although the planar boundary layer at Mach 8 would also be expected to be dominated by second-mode disturbances, this was not the case.[7] The major disturbances in the laminar planar boundary layer were low-frequency disturbances that were growing in a frequency band that was expected to be stable. These Mach 8 planar results appeared similar to the planar results obtained by Kendall[8] at Mach numbers of 3.0, 4.5, and 5.6 and by Demetriades[9] at $M = 3$. Thus, the instability phenomena producing planar boundary-layer transition in a conventional Mach 8 wind tunnel are different from what was anticipated based on guidance from classical linear stability theory, and it appears that this situation also exists at lower Mach numbers. Some of the confusion associated with hypersonic transition data probably has resulted from the fact it has been incorrectly assumed that hypersonic planar transition was a case of second-mode-dominated transition.

Referring back to Fig. 1, the hypersonic data that had the opposite trend of those expected from stability theory are flat-plate data. Since these are transition data, without any boundary-layer stability information, it is not possible to identify the instability phenomena producing transition. However, in view of the previously mentioned planar instability phenomena, it appears unlikely that second-mode disturbances were significant in these planar laminar boundary layers.

This is one more example of how easy it is to misinterpret transition data. To minimize future misinterpretations, more emphasis should be given to boundary-layer stability experimentation. Through stability experiments a better understanding of instability phenomena and thus a better understanding of transition are obtained.

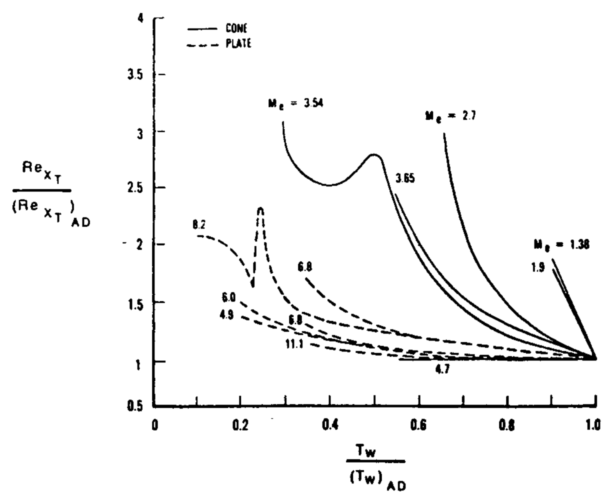

Fig. 1 Effect of surface cooling on transition (from Potter[3]).

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